

CONFORMALITY, PARTICLE PHENOMENOLOGY AND THE COSMOLOGICAL CONSTANT

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Abstract

Conformality is the idea that at TeV scales enrichment of the standard model particle spectrum leads to conformal invariance at a fixed point of the renormalization group. Some aspects of conformality in particle phenomenology and cosmology are discussed.

Alternative to “Grand” Unification

In GUT theories there is an unexplained hierarchy between the GUT scale and the weak scale which is about 14 orders of magnitude. There is the question of why these very different scales exist and how are the scales stabilized under quantum corrections?

Supersymmetry solves the second of these problems but not the first. Supersymmetry has some *successes*: (i) the cancellation of some UV divergences; (ii) the technical naturalness of the hierarchy; (iii) the unification of the gauge couplings; and (iv) its natural appearance in string theory.

On the other side, supersymmetry definitely presents several *puzzles*: (i) the “mu” problem - why is the Higgs at the weak scale not at the GUT scale?; (ii) breaking supersymmetry leads to too large a cosmological constant; and (iii) is supersymmetry really fundamental for string theory since there are solutions of string theory without supersymmetry.

These general considerations led naturally to the suggestion [1, 2, 3, 4, 5, 6] that supersymmetry and grand unification should be replaced by conformality at the TeV scale. Here it will be shown that this idea is possible, including explicit examples containing the standard model states. Further it will be shown that conformality is a much more rigid constraint than supersymmetry. Conformality predicts additional states at the TeV scale and a rich inter-family structure of Yukawa couplings.

Conformality as a Hierarchy Solution

First we note that quark and lepton masses, the QCD scale and weak scale are small compared to a (multi-) TeV scale. At the higher scale they may be put to zero, suggesting the addition of further degrees of freedom to yield a quantum field theory with conformal invariance. This has the virtue of possessing naturalness in the sense of 't Hooft [7] since zero masses and scales increases the symmetry.

The theory is assumed to be given by the action:

$$S = S_0 + \int d^4x \alpha_i O_i \quad (1)$$

where S_0 is the action for the conformal theory and the O_i are operators with dimension below four which break conformal invariance softly.

The mass parameters α_i have mass dimension $4 - \Delta_i$ where Δ_i is the dimension of O_i at the conformal point.

Let M be the scale set by the parameters α_i and hence the scale at which conformal invariance is broken. The for $E \gg M$ the couplings will not run while they start running for $E < M$. To solve the hierarchy problem we assume M is near to the TeV scale.

d = 4 CFTs

In enumerating the CFTs in 4 spacetime dimensions, we must choose the N of $SU(N)$. To leading order in $1/N$, the RG β -functions always vanish as they coincide with the $\mathcal{N} = 4$ case [8, 9]. For finite N the situation is still under active investigation. To *prove* the β - functions vanish when $\mathcal{N} = 0$ is rendered more difficult by the fact that without supersymmetry the associated nonrenormalization theorems are absent.

We extract the candidates from compactification[10] of the Type IIB superstring on $AdS_5 \times S^5/\Gamma$.

Let $\Gamma \subset SU(4)$ denote a discrete subgroup of $SU(4)$. Consider irreducible representations of Γ . Suppose there are k irreducible representations R_i , with dimensions d_i with $i = 1, \dots, k$. The gauge theory in question has gauge symmetry

$$SU(Nd_1) \times SU(Nd_2) \times \dots SU(Nd_k) \quad (2)$$

The fermions in the theory are given as follows. Consider the 4 dimensional representation of Γ induced from its embedding in $SU(4)$. It may or may not be an irreducible representation of Γ . We consider the tensor product of **4** with the representations R_i :

$$\mathbf{4} \otimes R_i = \oplus_j n_i^j R_j \quad (3)$$

The chiral fermions are in bifundamental representations

$$(1, 1, \dots, \mathbf{Nd}_i, 1, \dots, \overline{\mathbf{Nd}_j}, 1, \dots) \quad (4)$$

with multiplicity n_i^j defined above. For $i = j$ the above is understood in the sense that we obtain n_i^i adjoint fields plus n_i^i singlet fields of $SU(Nd_i)$.

Note that we can equivalently view n_i^j as the number of trivial representations in the tensor product

$$(\mathbf{4} \otimes R_i \otimes R_j^*)_{trivial} = n_i^j \quad (5)$$

The asymmetry between i and j is manifest in the above formula. Thus in general we have $n_i^j \neq n_j^i$ and so the theory in question is in general a chiral theory. However if Γ is a real subgroup of $SU(4)$, i.e. if $\mathbf{4} = \mathbf{4}^*$ as far as Γ representations are concerned, then we have by taking the complex conjugate:

$$n_i^j = (\mathbf{4} \otimes R_i \otimes R_j^*)_{trivial} = (\mathbf{4} \otimes R_i \otimes R_j^*)^*_{trivial} = (\mathbf{4}^* \otimes R_i^* \otimes R_j)_{trivial} = (\mathbf{4} \otimes R_i^* \otimes R_j)_{trivial} = n_j^i. \quad (6)$$

So the theory is chiral only if $\mathbf{4}$ is a complex representation of Γ , i.e. only if $\mathbf{4} \neq \mathbf{4}^*$ as a representation of Γ . If Γ were a real subgroup of $SU(4)$ then $n_i^j = n_j^i$.

If Γ is a complex subgroup, the theory is chiral, but it is free of gauge anomalies. To see this note that the number of chiral fermions in the fundamental representation of each group $SU(Nd_i)$ plus Nd_i times the number of chiral fermions in the adjoint representation is given by

$$\sum_j n_i^j Nd_j = 4Nd_i \quad (7)$$

(where the number of adjoints is given by n_i^i). Similarly the number of anti-fundamentals plus Nd_i times the number of adjoints is given by

$$\sum_j n_j^i Nd_j = \sum_j Nd_j (\mathbf{4} \otimes R_j \otimes R_i^*)_{trivial} = \sum_j Nd_j (\mathbf{4}^* \otimes R_j^* \otimes R_i)_{trivial} = 4Nd_i \quad (8)$$

Thus, comparing with Eq.(7) we see that the difference of the number of chiral fermions in the fundamental and the antifundamental representation is zero (note that the adjoint representation is real and does not contribute to anomaly). Thus each gauge group is anomaly free. The requirement of anomaly cancellation is, of course, a familiar one in string theory [12, 13] as well as in model building beyond the standard model [14, 15, 16, 17].

In addition to fermions, we have bosons, also in the bifundamental representations. The number of bosons M_i^j in the bifundamental representation of $SU(Nd_i) \otimes SU(Nd_j)$ is given by the number of R_j representations in the tensor product of the representation $\mathbf{6}$ of $SU(4)$ restricted to Γ with the R_i representation. Note that since $\mathbf{6}$ is a real representation we have

$$M_i^j = (\mathbf{6} \otimes R_i \otimes R_j^*)_{trivial} = (\mathbf{6} \otimes R_i^* \otimes R_j)_{trivial} = M_j^i$$

In other words for each M_i^j we have a *complex* scalar field in the corresponding bifundamental representation, where complex conjugation will take us from the fields labeled by M_i^j to M_j^i .

The fields in the theory are naturally summarized by a graph, called the quiver diagram [11], where for each gauge group $SU(Nd_i)$ there corresponds a node in the graph, for each chiral fermion in the representation (Nd_i, \overline{Nd}_j) , n_i^j in total, corresponds a directed arrow from the i -th node to the j -th node, and for each complex scalar in the bifundamental of $SU(Nd_i) \times SU(Nd_j)$, M_i^j in total, corresponds an *undirected* line between the i -th node and the j -th node

Interactions. Gauge fields interact according to gauge coupling which, combined with corresponding theta angle for i th group, is writtable as

$$\tau_i = \Theta_i + \frac{i}{4\pi g_i^2} = \frac{\tau d_i}{|\Gamma|}$$

where τ is complex parameter (independent i) and $|\Gamma| = \text{order } \Gamma$.

Yukawa interactions. Triangles in quiver. Two directed fermion sides and an undirected scalar side.

$$S_{Yukawa} = \frac{1}{4\pi g^2} \sum d^{abc} Tr \Psi_{ij}^a \Phi_{jk}^b \Psi_{ki}^c$$

in which d^{abc} is ascertainable as Clebsch-Gordan coefficient from product of trivial representations occurring respectively in $(4 \otimes R_i \otimes R_j^*)$, $(6 \otimes R_j \otimes R_k^*)$ and $(4 \otimes R_k \otimes R_i^*)$.

Quartic scalar interactions. Quadrilaterals in quiver. Four undirected sides. The coupling computable analagously to above.

Conformality. To leading order in $1/N$ all such theories are conformal[8, 9].

Are they conformal for higher orders?

YES, for $\mathcal{N} = 2$. All such $\mathcal{N} = 2$ theories are obtainable.

YES, for $\mathcal{N} = 1$: non-renormalization theorems ensure flat direction(s).

UNKNOWN for $\mathcal{N} = 0$.

Conformality for $\mathcal{N} = 0$. We can offer a plausibility argument for a conformal IR fixed point. If only one independent coupling occurs then the S-duality of the progenitor Type IIB superstring implies $g \rightarrow 1/g$ symmetry. If the next to leading order in $1/N$ is asymptotically free then IR flow increases g . Therefore for large g IR flow decreases g . Hence $\beta_g = 0$ for some intermediate g .

Applications of Conformality to Particle Phenomenology.

It is assumed that the Lagrangian is *nearly* conformal. That is, it is the soft-breaking of a conformal theory.

The soft breaking terms would involve quadratic and cubic scalar terms, and fermion mass terms. In the quiver diagram, these correspond respectively to 2-gons and triangles with undirected edges, and 2-gons with compatibly directed edges.

$$S = S_0 + \int \alpha_{ab} Tr \Psi_{1j*}^a \Psi_{ji*}^b + \alpha_{cd}^2 Tr \Phi_{1j*}^c \Phi_{ji*}^d$$

$$+\alpha_{efg}Tr\Phi_{ij*}^c\Phi_{jk*}^f\Phi_{ki*}^g+c.c.$$

Depending on the sign of the scalar mass term the conformal breaking could induce gauge symmetry breaking.

Consider a gauge subgroup $SU(Nd_i) \times SU(Nd_j)$ and suppose that $\langle \Phi_{ij*} \rangle \neq 0$. Assume for simplicity that $d_i = d_j = d$. Then the VEV can be represented by a square matrix with diagonal entries. The symmetry breaking depends on the eigenvalues. If there are two equal eigenvalues and the rest zero we get:

$$SU(Nd) \times SU(Nd) \rightarrow \\ SU(2)_{diagonal} \times U(1) \times SU(Nd-2) \times SU(Nd-2)$$

With more such VEVs and various alignments thereof a rich pattern of gauge symmetry breakings can emerge.

GENERAL PREDICTIONS.

Consider embedding the standard model gauge group according to:

$$SU(3) \times SU(2) \times U(1) \subset \bigotimes_i SU(Nd_i)$$

Each gauge group of the SM can lie entirely in a $SU(Nd_i)$ or in a diagonal subgroup of a number thereof.

Only bifundamentals (including adjoints) are possible. This implies no $(8, 2)$, etc. A conformality restriction which is new and satisfied in Nature!

No $U(1)$ factor can be conformal and so hypercharge is quantized through its incorporation in a non-abelian gauge group. This is the “conformality” equivalent to the GUT charge quantization condition in *e.g.* $SU(5)$!

Beyond these general consistencies, there are predictions of new particles necessary to render the theory conformal.

The minimal extra particle content comes from putting each SM gauge group in one $SU(Nd_i)$. Diagonal subgroup embedding *increases* number of additional states.

Number of fundamentals plus Nd_i times the adjoints is $4Nd_i$. Number N_3 of color triplets and N_8 of color octets satisfies:

$$N_3 + 3N_8 \geq 4 \times 3 = 12$$

Since the SM has $N_3 = 6$ we predict:

$$\Delta N_3 + 3N_8 \geq 6$$

The additional states are at TeV if conformality solves hierarchy. Similarly for color scalars:

$$M_3 + 3M_8 \geq 6 \times 3 = 18$$

The same exercise for $SU(2)$ gives $\Delta N_2 + 4N_3 \geq 4$ and $\Delta M_2 + 2M_3 \geq 11$ respectively.

FURTHER PREDICTIONS

Yukawa and Quartic interactions are untouched by soft-breaking terms. These are therefore completely determined by the IR fixed point parameters. So a rich structure for flavor is dictated by conformal invariance. This is to be compared with the MSSM (or SM) where the Yukawa couplings are free parameters.

GAUGE COUPLING UNIFICATION

Above the TeV scale couplings will not run. The couplings are nevertheless related, and not necessarily equal at the conformal scale.

For example, with equal $SU(Nd)$ couplings embed $SU(3)$, $SU(2)$, and $U(1)$ diagonally into 1, 3, 6 such groups respectively to obtain proximity to the correct ratios of the low-energy SM gauge couplings.

Some illustrative examples of model building using conformality:

We need to specify an embedding $\Gamma \subset SU(4)$.

Consider Z_2 . It embeds as $(-1, -1, -1, -1)$ which is real and so leads to a non-chiral model.

Z_3 . One choice is $\mathbf{4} = (\alpha, \alpha, \alpha, 1)$ which maintains N=1 supersymmetry. Otherwise we may choose $\mathbf{4} = (\alpha, \alpha, \alpha^2, \alpha^2)$ but this is real.

Z_4 . The only N = 0 complex embedding is $\mathbf{4} = (i, i, i, i)$. The quiver is as shown on the next transparency with the $SU(N)^4$ gauge groups at the corners, the fermions on the edges and the scalars on the diagonals. The scalar content is too tight to break to the SM.

Naming the nodes 0, 1, 2, 3, 4 we identify 0 with color and the diagonal subgroups (1,3) and (2,4) with weak and hypercolor respectively. There are then three families in

$$(3, \bar{3}, 1) + (1, 3, \bar{3}) + (\bar{3}, 1, 3)$$

and one anti-family.

We suppose that the soft conformal breaking excludes a mass term marrying the third family to its mirror.

There are sufficient scalars to break to the SM with three families.

This is an existence proof.

Simplest three family model has $\mathcal{N} = 1$ supersymmetry.

$$Z_3 \cdot 4 = (\alpha, \alpha, \alpha, 1)$$

Fermions and scalars are:

$$\sum_{i=1}^3 (3_i, \bar{3}_{i+1}) + \sum_{i=1}^3 (8+1)_i$$

$$\beta_g^{(1)} = -\frac{g^3}{16\pi^2} \left[\frac{11}{3} C_2(G) - \frac{2}{3} T(R_W) - \frac{1}{6} T(R_R) \right]$$

Find:

$$\beta_g^{(1)} \sim 9 - 9 = 0$$

for all three $SU(3)$ factors in supersymmetric trinification.

NON-ABELIAN ORBIFOLDS

We consider all non-abelian discrete groups up to order $g < 32$. There are exactly 45 such groups. Because the gauge group arrived at is $\otimes_i SU(Nd_i)$ we can arrive at $SU(4) \times SU(2) \times SU(2)$ by choosing $N = 2$.

To obtain chiral fermions one must have $\mathbf{4} \neq \mathbf{4}^*$ This is not quite sufficient because for $N = 2$, if $\mathbf{4}$ is complex but pseudoreal, the fermions are still non-chiral [6].

This last requirement eliminates many of the 45 candidate groups. For example $Q_{2n} \subset SU(2)$ has irreps of appropriate dimensions but cannot sustain chiral fermions. because these irreps are , like $SU(2)$, pseudoreal.

This leaves 19 possible non-abelian Γ with $g \leq 31$, the lowest order being $g = 16$. This gives only two families.

The smallest group which allows three chiral familes has order $g = 24$ so we now describe this model.

Using only D_N , Q_{2N} , S_N and T

(T = tetrahedral S_4/Z_2) one already finds 32 of the 45 non-abelian discrete groups with $g \leq 31$:

g	
6	$D_3 \equiv S_3$
8	$D_4, Q = Q_4$
10	D_5
12	D_6, Q_6, T
14	D_7
16	$D_8, Q_8, Z_2 \times D_4, Z_2 \times Q$
18	$D_9, Z_3 \times D_3$
20	D_{10}, Q_{10}
22	D_{11}
24	$D_{12}, Q_{12}, Z_2 \times D_6, Z_2 \times Q_6, Z_2 \times T$ $Z_3 \times D_4, Z_3 \times Q, Z_4 \times D_3, S_4$
26	D_{13}
28	D_{14}, Q_{14}
30	$D_{15}, D_5 \times Z_3, D_3 \times Z_5$

The remaining 13 of the 45 non-abelian discrete groups with $g \leq 31$ are twisted products:

g	
16	$Z_2 \tilde{\times} Z_8$ (two, excluding D_8), $Z_4 \tilde{\times} Z_4$ $Z_2 \tilde{\times} (Z_2 \times Z_4)$ (two)
18	$Z_2 \tilde{\times} (Z_3 \times Z_3)$
20	$Z_4 \tilde{\times} Z_5$
21	$Z_3 \tilde{\times} Z_7$
24	$Z_3 \tilde{\times} Q, Z_3 \tilde{\times} Z_8, Z_3 \tilde{\times} D_4$
27	$Z_9 \tilde{\times} Z_3, Z_3 \tilde{\times} (Z_3 \times Z_3)$

Successful $g = 24$ model is based on the group $\Gamma = Z_3 \times Q$.

The fifteen irreps of Γ are

$$1, 1', 1'', 1''', 2,$$

$$1\alpha, 1'\alpha, 1''\alpha, 1'''\alpha, 2\alpha,$$

$$1\alpha^{-1}, 1'\alpha^{-1}, 1''\alpha^{-1}, 1'''\alpha^{-1}, 2\alpha^{-1}.$$

The same model occurs for $\Gamma = Z_3 \times D_4$. The multiplication table is shown below.

	1	1'	1''	1'''	2
1	1	1'	1''	1'''	2
1'	1'	1	1'''	1''	2
1''	1''	1'''	1	1'	2
1'''	1'''	1''	1'	1	2
2	2	2	2	2	$1 + 1'$ $1'' + 1'''$
1α	1α	$1'\alpha$	$1''\alpha$	$1'''\alpha$	2α
$1'\alpha$	$1'\alpha$	1α	$1'''\alpha$	$1''\alpha$	2α
$1''\alpha$	$1''\alpha$	$1'''\alpha$	1α	$1'\alpha$	2α
$1'''\alpha$	$1'''\alpha$	$1''\alpha$	$1'\alpha$	1α	2α
2α	2α	2α	2α	2α	$1\alpha + 1'\alpha$ $1''\alpha + 1'''\alpha$

etc.

The general embedding of the required type can be written:

$$\mathbf{4} = (1\alpha^{a_1}, 1'\alpha^{a_2}, 2\alpha^{a_3})$$

The requirement that the **6** is real dictates that

$$a_1 + a_2 = -2a_3$$

It is therefore sufficient to consider for $\mathcal{N} = 0$ no surviving supersymmetry only the choice:

$$\mathbf{4} = (1\alpha, 1', 2\alpha)$$

It remains to derive the chiral fermions and the complex scalars using the procedures already discussed (quiver diagrams).

$D_4 \times Z_3$ MODEL.

VEVs for these scalars allow to break to the following diagonal subgroups as the only surviving gauge symmetries:

$$SU(2)_{1,2,3} \longrightarrow SU(2)$$

$$SU(2)_{5,6,7} \longrightarrow SU(2)$$

$$SU(4)_{1,2} \longrightarrow SU(4)$$

This spontaneous symmetry breaking leaves the Pati-Salam type model:

$$SU(4) \times SU(2) \times SU(2)$$

with three chiral fermion generations

$$3 [(4, 2, 2) + (\bar{4}, 2, 2)]$$

Towards the Cosmological Constant.

INCLUSION OF GRAVITY.

The CFT arrived at is in a flat spacetime background which does not contain gravity.

One way to introduce the four-dimensional graviton introduces an extra spacetime dimension and truncates the range of the fifth dimension. The four-dimensional graviton then appears by compactification of the higher-dimensional graviton, as is certainly the path suggested by the superstring.

Although conformality solves the hierarchy between the weak scale and the GUT scale, the hierarchy existing in non-string theory without gravity, it is clear that classical gravity violates conformal invariance because of its dimensional Newton coupling constant. The inclusion of gravity in the conformality scheme most likely involves a change in the spacetime at the Planck scale; one possibility being explored is noncommutative spacetime coordinates [18]. Another even more radical idea is the one already mentioned to invoke [19] at TeV scales an extra spacetime coordinate.

SUMMARY.

Conformality is seen to be a rigid organizing principle. Many embeddings remain to be studied. Soft breaking of conformal symmetry deserves further study, as does the even more appealing case of spontaneous breaking of conformal symmetry.

The latter could entail flat directions even in the absence of supersymmetry and if this is really possible one would need to invoke a symmetry different from supersymmetry to generate the flat direction.

This would lead naturally to an explanation of the vanishing cosmological constant different from any where a fifth spacetime dimension is invoked [20, 21].

New particles await discovery at the TeV scale if the conformality idea is valid.

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